COMPARISON OF ASYMMETRIC GARCH MODELS WITH ARTIFICIAL NEURAL NETWORK FOR STOCK MARKETS PREDICTION, A CASE STUDY

Samreen Fatima*, Mudassir Uddin

ABSTRACT

Much efforts have been done for modeling of financial data theoretically and empirically for the international stock markets, for example: Asia, Europe and Australia etc. But no frequent research has been done for the SAARC countries stock markets. Therefore, bench mark Index of Pakistan; Karachi Stock Exchange (KSE-100) and Bombay Stock Exchange (BSNSE) of India are selected as case study. They are not only the member of SAARC but also sharing the common border, due to this they are also involving in bilateral trading. We used closing indices of daily share price for the period of 1st January, 2010 to 15th January 2016. This study compares the forecasting performance and also investigates more volatile stock markets using Asymmetric GARCH (A-GARCH) models and non-parametric method (Artificial Neural Networks). In the A-GARCH; EGARCH and PGARCH models are used. Firstly, suitable Asymmetric GARCH (A-GARCH) model was developed for forecasting and investigating leverage effect. Secondly, an Artificial Neural Networks model was developed for the said stock markets. Lastly, forecasting performance of the FA-GARCH and ANN models both in and out sample were evaluated using root mean square error. In the A-GARCH; EGARCH (1,1) performed better than PGARCH(1,1) in both stock market data. However, when comparing A-GARCH with ANN, it was found that ANN gave minimum out sample forecasting error as compared to A-GARCH models. Therefore, ANN out played other studied models.

KEYWORDS: A-GARCH (Asymmetric GARCH), EGARCH (Exponential GARCH), PGARCH (Power GARCH), ANN (Artificial neural networks).

INTRODUCTION

Today countries all around the world are engaged to develop their social and economical relationship. Therefore, the South Asian Association for Regional Cooperation (SAARC) is one of the examples of such internationalism. It was formed in December 1985. Initially, it had seven members namely; Bangladesh, Bhutan, India, Maldives, Nepal, Pakistan, and Sri Lanka. But in 2005, after inclusion of Afghanistan now it has eight members. A treaty of South Asian Free Trade Area (SAFTA) was signed on January 6, 2004 at Islamabad between the initially joined member countries. The objective of the treaty was to promote and enhance the mutual trade and economic cooperation among the members and eliminate the barriers. Beside the constituents of SAARC; Pakistan and India are heavily populated countries and the sum of their economy is the largest economy in this region. Both have a great potential of bilateral trade because they are sharing common border, having same traits and socio-economy background. Currently, the trades between them are taking place by using the following means. Firstly, conventional trading by using undersigned official channels. Secondly, unofficial means through smuggling by using Indo-Pak land borders and through Afghanistan route. Finlay; via third countries such as Dubai and Singapore, their free ports are one of the means to transfer agents and traders from both countries.

Volatility clustering and leptokurtosis for the financial data has been observed by Mandelbrot1. Another important characteristic for financial time series is Leverage effect discussed Black2,3. Engle has characterized the changing in variances using the Autoregressive Conditional Heteroscedasticity (ARCH) class of models for conditional variance further extended in Generalized Autoregressive Conditional Heteroscedasticity (GARCH) by T. Bollerslev4. Simple GARCH models are not able to capture the leverage/ asymmetric effects in the stock returns because it has symmetric response between the returns and its volatility. Therefore, volatile model which pointed out the evidence of asymmetric responses proposed Nelson5.

Furthermore, GJR model6, the Asymmetric Power ARCH (APARCH) model by Ding et al7 and threshold

*University of Karachi, Pakistan
GARCH (TGARCH) model Zakoian⁹, all capture asymmetry properties in the returns. Samreen⁹ used ARCH-M model for benchmark index of Pakistan.¹⁰ Zafar Forecasting exchange rates of Pakistani rupees against UK pond using Bayesian forecasting method.

Artificial Neural Network is one of the dominant data mining techniques for prediction of daily closing prices because it depends on several known and unknown factors. ANN belongs to the family of non-linear and non-parametric models because they learn based on past and present data and predict future. It has been applied in a wide range of time series forecasting problems such as: financial data, forecasting of GDP, electricity prices, breast cancer, rainfall-runoff, scheduling policy system for flexible manufacturing systems.¹¹⁻¹⁷

Samreen et al compared the forecasting performance of daily returns of KSE100 index using ARIMA, ARCH/GARCH and Artificial Neural Networks models and found that Artificial Neural Networks perform well as compared to ARIMA and ARCH/GARCH. A hybrid financial system of KSE-100 index developed Samreen et al,¹⁹ in their developed system first they used GARCH model to capture volatility in the returns and then the estimates of volatile model was given as input to ANN model. Their proposed hybrid financial system outplayed as compared to simple GARCH and ANN model.

The layout of this research paper is as follows. Introduction of A-GARCH model is defined in Section 2. Section 3 discusses the brief introduction of Artificial Neural Network (ANN) method. Section 4 deals with data analysis of A-GARCH models and ANN models. Section 5 consist of conclusion.

INTRODUCTION OF ASYMMETRIC GARCH

EGARCH model

In simple GARCH model, squared residuals enter in the variance equation which does not captures successfully the fat tail behavior and volatility clustering properties. There is a stylized fact of the returns that negative shocks have increased the volatility than good news. In other words, a falling market may increase the volatility than a rising market. This news impact asymmetric behavior is commonly referred to as the leverage effect Zivot. Nelson⁷ proposed EGARCH model in 1991 in which forecast values are always positive because the left hand side of the equation is the Logarithmic transformation of conditional variance. Variable under analysis depends on both size and the sign of the lagged residuals. σ² is an asymmetric function of past ε’s is defined ε̃ = zσ̃.

Where z is an independently and identically (i.i.d) process with E[z] = 0 and V[z] = 1 and σ time varying but positive and measurable function of the information set at time t-1. is serially uncorrelated with mean zero and the. EGARCH (1,1) model is given by eq 1.

\[
\log \sigma^2_t = \omega + \alpha \frac{|\varepsilon_{t-1}|}{\sigma^2_{t-1}} + \gamma \frac{\varepsilon_{t-1}}{\sigma^2_{t-1}} - \beta \log \sigma^2_{t-1}
\]  

The generalized form of EGARCH (p,q) model is described by eq 2,

\[
\log \sigma^2_t = \omega + \sum_{i=p}^{p} \alpha_i \frac{|\varepsilon_{t-i}|}{\sigma^2_{t-i}} + \sum_{i=q}^{q} \beta_i \frac{\varepsilon_{t-i}}{\sigma^2_{t-i}} - \beta \log \sigma^2_{t-1}
\]

ε follows a normal standard distribution, the parameters of equation (2) ω, α, and β are not restricted to be nonnegative and ∑α_i + ∑β_i = 1. The parameter of equation (2) captures the leverage effect, negative value of gamma indicating the relationship between volatility and their return has negative effect (bad news). On the other hand positive shocks (good news) have less effect on the conditional variance as compared to the negative shocks.

PGARCH model

Power Generalized Autoregressive Conditional Heteroscedastic GARCH (PGARCH) which also deals with leverage effects developed Ding et al. The generalized form of PGARCH can be defined by eq 3;

\[
\sigma^2_t = \alpha_0 + \sum_{i=1}^{q} \beta_i \sigma^2_{t-i} + \sum_{i=1}^{q} \alpha_i \left( \varepsilon_{t-i} - \gamma \varepsilon_{t-i} \right) \delta
\]

and the P-GARCH (1,1) is described by eq 4:

\[
\sigma^2_t = \alpha_0 + \beta \sigma^2_{t-1} + \alpha_i \left( \varepsilon_{t-i} - \gamma \varepsilon_{t-i} \right) \delta
\]

In equation (4) α, and β are the standard GARCH parameters, but parameter γ measure the leverage effect. δ is the power parameter offers the opportunity to model the conditional variance and standard deviation which
cannot be possible with another asymmetric GARCH model. In the general PGARCH model if $\delta=2$, and $\gamma=0$ then it becomes a standard GARCH model and if $\delta=1$, then from the above equation conditional standard deviation will be estimated.

**ARTIFICIAL NEURAL NETWORK**

An Artificial Neural Networks (ANN) is one of the fastest growing technology inspired by the biological network. ANN working principles are similar to the neural systems of human brain. ANN is very powerful non-linear and non-parametric tool with high degree of accuracy for the real world problem. It is being applied in many fields such as: computer science, medical diagnosis, robotics, astronomy, pattern classification, and also used for modeling and forecasting of financial time series. It is based on flexible computing frameworks and universal approximations which can be applied to a wide range of forecasting problems Mehdi et al.21. It does not need any prior information for model building process because it learns through experiences.

Feed forward network of an input layer, an output layer and a single hidden layer is widely used for time series modeling and forecasting Zhang22. A perceptron is the simplest form of network which consists only two layers: an input layer of source nodes that projects onto an output layer of neurons. In this study the focus is multilayer feed-forward neural network, which contains an input layer, receives information from external sources; one or more hidden layers, acting as intermediate computational layers; and an output layer, results of input layer.

Mostly, financial data are non stationary but can be converted into stationary such as lag difference one more times or logarithmic transformation of non stationary process. So, in ANN if input nodes are lagged values then functional form ANN can be defined as,

$$y_t = h(y_{t-1},..., y_{t-p}, \alpha) + \epsilon_t \quad (5)$$

or above functional form can be expressed mathematically.

$$y_t = a_0 + \sum_{i=1}^{p} a_{i} y_{t-i} + \sum_{k=1}^{h} a_{h} h \left( \sum_{k=1}^{h} a_{k} y_{t-k} + \sum_{k=1}^{h} a_{k} y_{t-k} \right) + \epsilon_t \quad (6)$$

From equation (6) $y_{t, i}$ transformed data (input), weights associated with input nodes $a_{ik}$, $\alpha$ weights associated with hidden nodes, is called bias of input, $k$ is the number of input nodes and $h$ is the number of hidden nodes and $h(\cdot)$ is a non linear activation function so $\sum_{k=1}^{h} a_{k} y_{t-k} + \sum_{k=1}^{h} a_{k} y_{t-k}$ is non linear. Activation functions are also called transfer function because they transfer input to the hidden layer and make them non-linear. The most widely used activation functions are logistic and hyperbolic functions. It has been observed that a simple network structure which has a small number of hidden nodes perform well in out-of-sample forecasting. Because too many hidden layers confuse the network, this may be causes over fitting. The network can be train by using small weights, usually weights are taken within the range $[-1,1]$. These weights are updated through learning process. There are different learning algorithms such as: Gradient descent, Back-propagation algorithms etc. In this study we used Back propagation algorithms which basically propagates errors back during the training process and weights are adjusted using these errors.

**DATA ANALYSIS AND RESULTS**

**Model building process of A-GARCH**

In this study we selected daily closing prices of Karachi and Bombay stock markets from 1st January, 2010 to 15th January 2016 total of 1576 excluding weekends. Data from 1st January, 2010 to 8th January, 2016 used for model building and from 9th January, 2016 to 15th January, 2016 sample kept as a holdback period in order to compare out sample good or bad forecasting performance.

We calculated returns of these indices using the transformation $y_t = (\log x_t - \log x_{t-1})$.

Figure 1(a) and 2(a) show time series plot of BSNSE and KSE-100 index. However, Figure 1(b) and 2(b) are logarithmic transformed data, showing leverage effect and volatility clustering.

Descriptive statistics of the returns are summarized in the Table 1 shows that both countries have positive mean returns indicating increase in price during the selected period. They are negatively skewed and show that there is a high probability of earning returns which
We know that EGARCH, and PGARCH models are capable for modeling asymmetric effect in the returns of the daily closing prices. Therefore, the selection of the suitable order of the FA-GARCH model is one of the major concerns. There are several traditional selection criterion such as: AIC, SBIC and log likelihood etc. In this study we used AIC and SBIC both criterion for selecting the suitable model. Different order of EGARCH and PGARCH models are built based on the definition of parsimonious model. Therefore, the order of the model ‘p and q’ is taken in such a manner 1 < P & q < 2. The parameters of the FA-GARCH models are estimated using the maximum likelihood method.

**Table 1: General statistics of closing price returns of the stock markets for the period 1st January, 2010 to 15th January, 2016.**

<table>
<thead>
<tr>
<th>Values</th>
<th>KSE</th>
<th>BSNSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.028681</td>
<td>0.085765</td>
</tr>
<tr>
<td>Median</td>
<td>0</td>
<td>0.061</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.703</td>
<td>4.419</td>
</tr>
<tr>
<td>Minimum</td>
<td>-6.12</td>
<td>-4.558</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>1.02768</td>
<td>0.895791</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.176767</td>
<td>-0.47312</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>4.648085</td>
<td>5.942218</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>178.4019</td>
<td>599.7864</td>
</tr>
</tbody>
</table>

It is found that EGARCH (1, 1) is suitable based on AIC and SBIC for both returns. The estimated coefficients of the models are statistically significant at 5% level of significance. The estimated results of the parameters with standard error and p-value of both models are given in table 2. Parameter $\gamma_1$ that is almost negative in both returns is greater than mean. The kurtosis for all series is much larger than its normal value reflecting the fact that the tails of the distributions of all series are fatter than the normal distribution. It is found that BSNSE is more fatter than KSE-100 which is confirmed by using Jarque-Bera test statistics at 5% level of significance. Therefore, the null hypothesis of normality is to be rejected. Moreover, Lagrange Multiplier test provide the evidence of hetroscedasticity in both returns.
markets shows the presence of asymmetric effect, which indicates that loses increase the volatility during the study period. The values are -0.1073 for BSNSE, and -0.1883 for KSE. Therefore, BSNSE is affected less by bad news then KSE-100 based on the estimated value of $\gamma$ (leverage parameter). In all daily closing indices, symmetric effect $\alpha$ is positive 0.0897 for BSNSE and 0.2322 for KSE 100 indicate that the KSE-100 is sensitive to the market events more than by BSNSE during the study period. However, the parameter $\beta$, that is greater than 0.9 shows the persistence in conditional volatility. So, BSNSE is more persistent than KSE-100 because the $\beta$ for BSNSE has 0.95 and 0.8790 for KSE-100.

Mean and variance equations of the BSNSE for EGARCH (1,1) model are:

$$y_t = 0.000215 + \varepsilon_t$$

$$\log \sigma_t^2 = -0.4643 + 0.089658 \left( \frac{\gamma_1}{\sqrt{\sigma_t^2}} \right) + 0.1073 \frac{\gamma_1}{\sqrt{\sigma_t^2}} + 0.9571 \log \sigma_t^2$$

Mean and variance equations of the KSE-100 for EGARCH (1,1) model are:

$$y_t = 0.000219 + \varepsilon_t$$

$$\log \sigma_t^2 = -1.3386 + 0.2322 \left( \frac{\gamma_1}{\sqrt{\sigma_t^2}} \right) + 0.1883 \frac{\gamma_1}{\sqrt{\sigma_t^2}} + 0.8790 \log \sigma_t^2$$

By fixing the power parameter 1 we found PGARCH (1,1) has minimum AIC and SBIC for both closing indices. Table 3, presents results of PGARCH (1,1). The coefficients are significant at 5% level of significance and parameter $\gamma_1$ measure the asymmetric effect has positive value in both data sets for the selected periods, increase the volatility due to negative shocks.

$$y_t = 0.000219 + \varepsilon_t$$

Table 2: Output of BSNSE and KSE-100 using EGARCH (1, 1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.0002</td>
<td>0.0002</td>
<td>0.9256</td>
<td>0.3500</td>
</tr>
<tr>
<td>$\omega$</td>
<td>-0.4643</td>
<td>0.0713</td>
<td>-6.5110</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.0897</td>
<td>0.0196</td>
<td>4.5733</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>-0.1073</td>
<td>0.0116</td>
<td>-9.2818</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.9571</td>
<td>0.0070</td>
<td>136.4202</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 3: Output of BSNSE and KSE-100 using PGARCH (1, 1)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>0.0011</td>
<td>0.0002</td>
<td>5.5980</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>0.00038</td>
<td>6.76E-05</td>
<td>5.6982</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.010962</td>
<td>4.77774</td>
<td>7.0638</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.999</td>
<td>0.215452</td>
<td>4.64138</td>
<td>0.0000</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>0.7778</td>
<td>0.02449</td>
<td>31.753</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
Mean and variance equations of the KSE-100 for EGARCH (1, 1) model is:

\[ \sigma_i^2 = 0.00038 + 0.9221\sigma_{i-1}^2 + 0.523(e_{i-1}^2) \]

Mean and variance equations of the KSE-100 for EGARCH (1, 1) model is:

\[ y_i = 0.001132 + \varepsilon_i \]

\[ \sigma_i^2 = 0.00101 + 0.7778\sigma_{i-1}^2 + 0.1356(e_{i-1}^2) \]

ANN model building process

Table 4: Error sum of squares validation sets and their corresponding out sample FRMSE of BSNSE and KSE-100

<table>
<thead>
<tr>
<th>Country</th>
<th>Error sum of square of validation sets</th>
<th>Out sample Forecast RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>KSE 100</td>
<td>0.000938</td>
<td>442.571</td>
</tr>
<tr>
<td></td>
<td>0.000405</td>
<td>284.5991</td>
</tr>
<tr>
<td></td>
<td>0.001608</td>
<td>572.125</td>
</tr>
<tr>
<td></td>
<td>0.000938</td>
<td>384.2289</td>
</tr>
<tr>
<td>BSESN</td>
<td>0.000247</td>
<td>172.983</td>
</tr>
<tr>
<td></td>
<td>0.000278</td>
<td>184.0285</td>
</tr>
<tr>
<td></td>
<td>0.000209</td>
<td>153.5415</td>
</tr>
<tr>
<td></td>
<td>0.000443</td>
<td>232.8541</td>
</tr>
<tr>
<td></td>
<td>0.000205</td>
<td>157.8324</td>
</tr>
</tbody>
</table>

KSE-100 has minimum validation error 0.000405 with training period 95 observation and 0.000209 has training period 108 data points for BSESN. Out sample forecast root mean square error was calculated using data set from 1st January, 2016 to 15th January, 2016 (table 4). Table 5, compares the performance of ANN and A-GARCH model using out sample forecast RMSE.

Table 5:  In and Out-sample forecast of BSNSE and KSE-100 of A-GARCH and ANN

<table>
<thead>
<tr>
<th>Country</th>
<th>Model</th>
<th>In sample RMSE</th>
<th>In sample MAE</th>
<th>In sample MAPE</th>
<th>Out sample RMSE</th>
<th>In sample RMSE</th>
<th>In sample MAE</th>
<th>In sample MAPE</th>
<th>Out sample RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>KSE 100</td>
<td>PGARCH(1,1)</td>
<td>192.7</td>
<td>125.2</td>
<td>0.63</td>
<td>419.2</td>
<td>192.7</td>
<td>125.1</td>
<td>0.63</td>
<td>419.2</td>
</tr>
<tr>
<td></td>
<td>EGARCH(1,1)</td>
<td>192.7</td>
<td>125.1</td>
<td>0.63</td>
<td>419.2</td>
<td>192.7</td>
<td>125.1</td>
<td>0.63</td>
<td>419.2</td>
</tr>
<tr>
<td></td>
<td>ANN</td>
<td></td>
<td></td>
<td></td>
<td>284.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BSESN</td>
<td>PGARCH(1,1)</td>
<td>208.9</td>
<td>153.8</td>
<td>0.75</td>
<td>187.2</td>
<td>208.9</td>
<td>153.8</td>
<td>0.75</td>
<td>187.1</td>
</tr>
<tr>
<td></td>
<td>EGARCH(1,1)</td>
<td>208.9</td>
<td>153.8</td>
<td>0.75</td>
<td>187.1</td>
<td>208.9</td>
<td>153.8</td>
<td>0.75</td>
<td>187.1</td>
</tr>
<tr>
<td></td>
<td>ANN</td>
<td></td>
<td></td>
<td></td>
<td>153.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In ANN modeling selection of number input layer, input nodes, hidden layers, output nodes, nonlinear functions, training algorithms, and weights are of primary concern. Data from 1st January, 2010 to 8th January 2016 used for model building and from 9th January, 2016 to 15th January, 2016 sample is used to compare out sample forecasting performance. Due to high volatility in the closing prices, a logarithmic transformation \( y_i = \log x_i - \log x_{i-1} \) was applied to the data set to smooth the volatility effect (make stationary). Where \( \{x_i\} \) is the closing price series of two stock markets. In this study, we used five input nodes, one hidden nodes and one output node that is 5-1-1 as the trading is open in stock market (from Monday to Friday) 5 days.

1st January, 2010 to 8th January 2016 data is divided into training and validation subsets. Training and validation are further divided into small groups in such a manner that first training set and then validation set, second training set and validation set similarly, last training set and validation set till all the groups are exhausted.

Each training group estimates the parameter and corresponding validation compute the error sum of squares (local minima) of the selected subset. Training of the network is started by taking small random weights that is in the range of \([-1, 1]\). This training process provides a set of error sum of squares.

In out sample FRMSE, asymmetric GARCH models results are almost same in both markets, but PGARCH (1,1) modeled conditional standard deviation rather than variance. However, KSE 100 and BSNSE have minimum out sample FRMSE in ANN as compared to PGARCH (1,1) and EGARCH(1,1).
CONCLUSION

This study has used Asymmetric GARCH models such as EGARCH and PGARCH that explain the leverage effect as it increases volatility that is an important characteristic of financial market. Therefore, more volatile market does not attract investor resulting weak economy and slow development of the country. It is found that Karachi stock market was more volatile during the selected study period than BSNSE based on the EGARCH and PGARCH model. In order to assess the forecasting performance of the models out sample forecast root mean squares error was used. Different values of ‘p’ and ‘q’ were applied by keeping the range of ‘p and q’ fix i.e. 1 < P & q < 2. EGARCH (1,1) and PGARCH(1,1) were found suitable as they have minimum in-sample RMSE and out sample FRMSE. In A-GARCH model, in-sample RMSE, MAE, MAPE and out sample FRMSE are approximately same in both types of A-GARCH models. Therefore, it is suggested that anyone A-GARCH model can be used for future forecasting. EGARCH model is preferred over PGARCH because of simplicity. Furthermore, ANN models were developed for both markets using different training periods and based on minimum error sum of square of validation sets suitable ANN model was selected for each market. As the Stock markets are non linear in behavior, ANN model fully capture this nonlinearity itself, no prior information is required. Therefore, empirical analysis shows, ANN outplayed as compared to both A-GARCH in term of out sample FRMSE.

REFERENCE


